Chapter 2
Review of Ice Processes and Properties

2-1. Introduction

Each year, ice grows on and disappears from the Nation's rivers and lakes in tune with the cycles of nature. Unless the ice causes problems, such as flooding or blocking the arteries of commerce, few people pay more than cursory attention. Because of this, the very great variety of ways in which ice initially forms, grows, and accumulates, and finally disappears, are relatively unknown to the casual observer. Ice processes on lakes are different from those on rivers. And for both lakes and rivers, the size of the water body affects which processes take place. Ice owes its existence to the thermal processes of phase change and heat transfer, but its evolution is greatly influenced by physical and mechanical processes. Thus, this chapter introduces the wide variety of ice formation, evolution, and destruction mechanisms and identifies the principal thermal, physical, and mechanical properties that govern them.

2-2. Physical Properties of Ice and Fresh Water

a. Water properties. In ice engineering, ice is most often encountered in contact with liquid water. Therefore, it is important to be aware of the physical properties of water to fully understand the interaction of ice and water. The physical properties of greatest importance are density and specific heat.

(1) Density. The density of water (mass per unit volume) is temperature-dependent, as are many of its physical properties. The changes in water density with temperature are relatively small over the normal range of water temperature, but these small changes can have large-scale results. The primary example of this is the *thermal stratification* of lakes and reservoirs. The density change of water in response to temperature is unusual compared to almost all other substances—the density of water does not increase continuously with decreasing temperature but has a maximum at 4°C (39.2°F). A further temperature decrease causes the density of water to decrease. This density maximum has a profound effect on the thermal stratification of lakes and reservoirs in winter. The density of water as a function of temperature is reasonably well described by the following equation (Heggen 1983)

$$\rho = 1000.0 - 1.955 \times 10^{-2} |\theta - 4.0|^{1.68}$$

where ρ is the density of water in kilograms per cubic meter (kg/m^3), and θ is the temperature in degrees Celsius. In English units the equation is

$$\rho = 1.940 - 1.413 \times 10^{-5} |\theta - 39.2|^{1.68}$$

where ρ is the density of water in slugs per cubic foot (slugs/ft^3) and θ is the temperature in degrees Fahrenheit. At 0°C (32°F), the density of water is 999.80 kg/m^3 (1.94 slugs/ft^3 or 62.42 lb/ft^3 specific weight).

(2) Specific heat. Specific heat is a measure of the quantity of heat required to raise the temperature of one unit mass of fluid one unit degree under constant pressure. The specific heat of water is much larger than the specific heat of most materials. As a result, a relatively large amount of heat must be added to or

extracted from water to change its temperature. The specific heat of water as a function of temperature is described by the following equation (Heggen 1983)

$$C_p = 4174.9 + 1.6659 \left(e^{r/10.6} + e^{-r/10.6} \right)$$

where $r = 34.5 - \theta$ for $\theta < 34.5°C$, θ being the temperature in degrees Celsius, and C_p is the specific heat in joules per kilogram per degree Celsius (J/kg °C). In English units

$$C_p = 0.99716 + 3.9789 \times 10^{-4} \left(e^{r/19.08} + e^{-r/19.08} \right)$$

where $r = 94.10 - \theta$ for $\theta < 94.10°F$, θ being the temperature in degrees Fahrenheit, and C_p is the specific heat in British Thermal Units per pound per degree Fahrenheit (Btu/lb °F). At 0°C (32°F), C_p = 4218.13 J/kg °C (1.0075 Btu/lb °F).

(3) Density stratification in natural water bodies. Stratification results from differences in density and temperature occurring in a vertical section of a lake or reservoir. Lighter fluids "float" on top of denser, heavier fluids. In the summer months, the temperature of the water in a lake or reservoir will be much greater than 4°C (39°F), and warmer water will float on top of colder, denser water. As a result, in summer the water near the surface will be warmer than the water found at depth. In the winter months, when the temperature of the water in the lake or reservoir is at 4°C (39°F) or less, the less dense water will be the colder water and this water will float on top of the warmer, denser water, which has a temperature closer to 4°C (39°F). As a result, in winter the water near the surface will be colder than the water found at depth. This warmer water found at depth in lakes and reservoirs forms a "thermal reserve." If available in sufficient quantities, this reserve can be used to melt ice at the water surface by bringing up the denser, warmer water using bubblers (see Chapter 3) or mechanical diffusers.

(4) Mixing. The density difference between 0 and 4°C (32 and 39°F) is small, and it does not take much mixing action to overcome the stratification. Turbulence is a very effective mixer. All rivers, streams, and channels with any appreciable flow velocity are turbulent and therefore will be well-mixed vertically and will exhibit virtually no stratification. Therefore, there is almost no thermal reserve located at depth in flowing rivers, streams, or channels that can be exploited for melting ice. Lakes and reservoirs may be well-mixed to some depth owing to the turbulence created by wind. Ponds and shallow lakes may be well-mixed throughout their entire depth during times when there are strong winds blowing. The presence of an intact ice cover will generally protect the water below from the influence of the wind and promote stratification.

b. Density of freshwater ice. The density of freshwater ice is 916.8 kg/m^3 at 0°C (1.779 slugs/ft^3 or 57.2 lb/ft^3 specific weight at 32°F). Like most materials, ice becomes denser with decreasing temperature (at –30°C [–22°F], the density of ice is about 920.6 kg/m^3 [1.786 slugs/ft^3 or 57.5 lb/ft^3 specific weight]). The density of ice is affected by the presence of impurities, with the two most common "impurities" being air bubbles and unfrozen water. The presence of air bubbles tends to reduce the density, and unfrozen water tends to increase the density. Unfortunately, for ice found on natural water bodies, little can be said about the amount of these "impurities" without resorting to direct and somewhat difficult measurements. As a result, for engineering calculations, the approximation for the density of ice of 915–917 kg/m^3 (1.775–1.779 slugs/ft^3 or 57.1–57.2 lb/ft^3 specific weight) is probably adequate.

c. Thermal properties. The thermal properties most often needed are the thermal conductivity of ice and the latent heat.

(1) Thermal conductivity. Thermal conductivity describes the ability of ice to transmit heat under a unit temperature gradient. The temperature dependence of thermal conductivity is described by

$$k_i = 2.21 - 0.011\theta$$

where k_i is the thermal conductivity in watts per meter per degree Celsius (W/m °C) and θ is the temperature in degrees Celsius. In English units

$$k_i = 1.27 - 0.0061\ (\theta - 32)$$

where k_i is the thermal conductivity in British Thermal Units feet per hour per square foot per degree Fahrenheit (Btu ft/[hr ft^2 °F]) and θ is the temperature in degrees Fahrenheit. The thermal conductivity of ice is greater than that of concrete (0.81–1.40 W/m °C {0.47–0.81 Btu ft/[hr ft^2 °F]}) and wood (0.14–0.21 W/m °C {0.08–0.12 Btu ft/[hr ft^2 °F]}) but much less than that of metal (for example, copper 388 [224], aluminum 209 [120], and steel 49 [3]). Ice is a not a great insulator, but it is not much of a heat conductor either. The thermal conductivity of ice is significantly influenced by air bubbles and the inclusion of unfrozen water. But as with the density determination, the amount of both of these impurities in ice in natural water bodies is usually not known, and as a result their influence is usually ignored.

(2) Latent heat. Pure water freezes at 0°C (32°F) under standard atmospheric pressure. When water freezes, 333.4 J/g (143.3 Btu/lb) of latent heat is released. This is a substantial amount of heat, especially when compared to the 4.217 J/g (1.813 Btu/lb) it takes to change the temperature of water 1°C (1.8°F).

2-3. Mechanical Properties of Freshwater Ice

Mechanical properties are important parameters that control the forces that ice may exert upon structures and the deformation of ice under load. Ice is a complex material whose behavior under load can range from brittle to ductile, depending on its structure, the rate of load application, and temperature. Because of these factors, the values of ice properties also vary with the measurement techniques and conditions. Only a brief summary of the mechanical properties of freshwater ice is presented below. The reader interested in ice rheology and ice mechanics should consult more specialized texts such as Pounder (1965), Michel (1978), or Ashton (1986).

a. Ice strength. Strength is defined as the maximum stress that a test specimen can support immediately before failure. Its value will thus depend on the mode of failure (e.g., bending or flexure, crushing or compression, shear), the type of failure (namely brittle or ductile), the presence of flaws in the ice, and, as already mentioned, the test technique. In the following, only brittle failure will be considered, since it corresponds to the relatively high loading rate more commonly associated with ice impact on structures when driven by water flow or wind.

(1) Bending or flexural strength. The ice bending or flexural strength is the maximum stress that an ice sheet or ice floe can withstand when subjected to a vertical load at the edge of the ice sheet, e.g., when riding up an inclined slope or striking an inclined bridge pier. A number of studies have measured ice

flexural strength (Frankenstein 1968, Lavrov 1969, Gow 1977). From these, the expected bending strength of competent, columnar freshwater ice ranges from a low of 0.5 MPa (70 psi) for relatively large specimens tested by the cantilever beam method to a high of 1.2 MPa (170 psi) for small, simple beam specimens. This range of values also reflects differences in results obtained depending on whether the tests were conducted with the top of the ice under tension or the bottom of the ice under tension and the corresponding variation in crystal size.

(2) Crushing or compressive strength. The ice compressive strength is the maximum stress before failure that ice can withstand when subjected to in-plane loads, i.e., normal to the ice floe thickness, as when being pushed against a vertical surface or bridge pier. The main factors that affect the crushing strength of ice are the crystal size, the rate of loading (strain rate), and the ice temperature. On the average, for columnar ice and snow or frazil ice at about –10°C (14°F) and in the brittle range of failure, i.e., for relatively high rate of loading, the crushing strength is in the range of 8 to 10 MPa (1.1 to 1.5 kpsi). Michel (1978) gives the following equation for estimating the ice crushing strength

$$\sigma = 9.4 \times 10^5 \left(d^{-1/2} + 3 \, |\theta|^{0.78} \right) \tag{2-1}$$

where

σ = crushing strength (pascals)

d = crystal size (centimeters)

θ = temperature (degrees Celsius).

(3) Breakthrough loads. The bearing capacity of ice is discussed in some detail in Chapter 8. For short-term duration loads, the allowable load P that a floating ice sheet can support is proportional to the square of the ice thickness h, that is

$$P = A \, h^2 . \tag{2-2}$$

For most practical purposes, the value for A can be taken as 1/100 when P is expressed in metric tons (1000 kg) and h in centimeters (A can be taken as 1 when P is expressed in meganewtons and h in meters), and for P in tons and h in inches, then A can be taken as 1/16.

b. Elastic modulus. The elastic modulus E describes the relationship between stress and strain. For the case of ice, the elastic modulus has been found to depend on the ice temperature, crystal structure, and the rate of stress application. Also, creep in ice can occur soon, especially at high stress levels, requiring that strain be measured "extremely quickly after the application of the stress" (Ashton 1986). As for the other mechanical properties of ice, the measured values of the elastic modulus also depend on the measurement techniques. As a result, estimates of the elastic modulus can range widely, and values estimated or measured in the field for the elastic modulus of intact freshwater ice range from about 0.4 to 9.8 GPa (55 to 1350 kpsi). The elastic modulus of ice grown in large laboratory tanks ranges from about 4.3 to 8.3 GPa (600 to 1150 kpsi), whereas the elastic modulus of small laboratory specimens is typically higher. Values for extensively cracked or deteriorated ice may be much lower.

c. Characteristic length. The characteristic length L_c of a floating ice sheet is a measure of the extent of the zone of deformation when the ice is subjected to a vertical load. It also governs the initial size of ice floes resulting from the breakup of a sheet ice cover. This parameter is expressed in terms of the ice thickness h and modulus of elasticity E by

$$L_c = \left[\frac{E h^3}{12\gamma \left(1 - \nu^2\right)}\right]^{1/4} \tag{2-3}$$

where γ is the specific weight of water and ν is the Poisson's ratio of ice, usually taken to be 0.3. From elastic analysis, the radius of the area of deformation is approximately equal to 3 characteristic lengths. Field measurements (Sodhi et al. 1985) have shown that the characteristic length of competent freshwater ice is about 15 to 20 times the ice thickness, with the higher ratio corresponding to cold ice and the lower values for warm ice in late winter or early spring.

d. Field measurements. There is no simple, reliable method to measure the compressive strength of ice in the field. It is often necessary to collect ice samples for testing in the laboratory under controlled conditions. The flexural strength and the elastic modulus can, on the other hand, be measured in the field with a minimum of equipment (IAHR 1980) using one of the techniques described below.

(1) Cantilever beam. A cantilever beam of length L (= 5 to 8 h) and width B ($\cong$ 2 h) is cut in the ice sheet (Figure 2-1a). A load P is applied to the tip of the beam and the corresponding deflection δ is measured. The elastic modulus E is given by

$$E = \frac{4}{B} \left(\frac{L}{h}\right)^2 \frac{P}{\delta}. \tag{2-4a}$$

If P' is the failure load of the cantilever beam, the flexural strength is calculated by

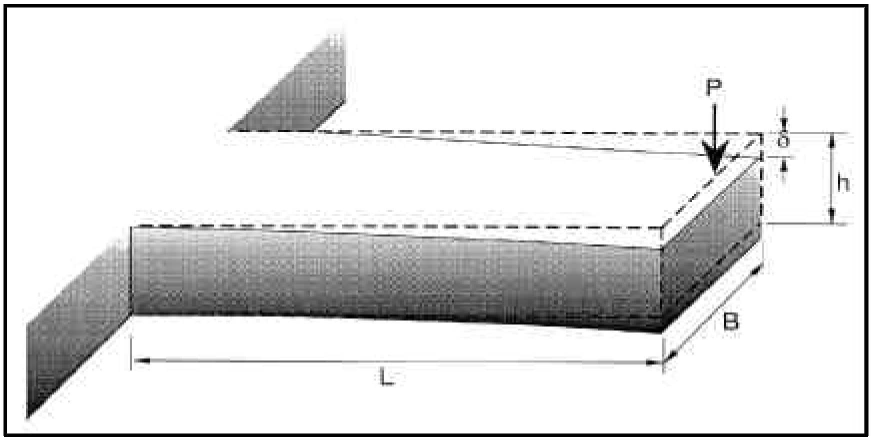
$$\sigma_b = 6 \frac{P'L}{Bh^2}. \tag{2-4b}$$

To make the results as reliable as possible, the saw cuts at the root of the cantilever beam should be rounded to avoid local stress concentration and resulting early failure of the beam.

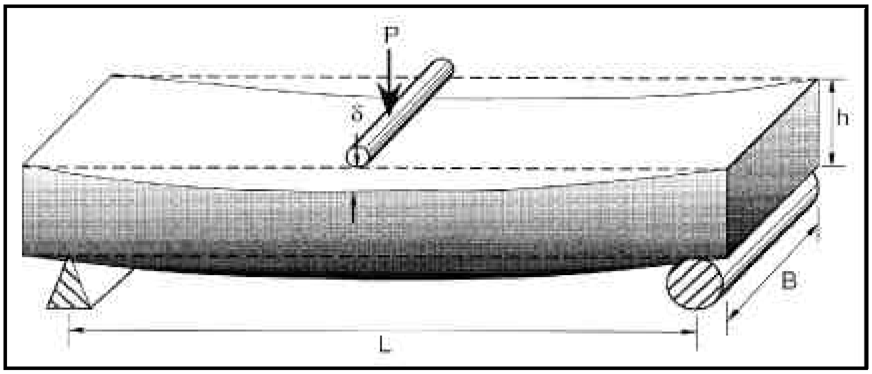
(2) Simple beam. A beam of length L and width B cut from the ice sheet is placed on two supports and loaded in the beam center with a load P that yields a deflection δ (Figure 2-1b). The corresponding value of the elastic modulus is

$$E = \frac{P}{4B\delta} \left(\frac{L}{h}\right)^3. \tag{2-5a}$$

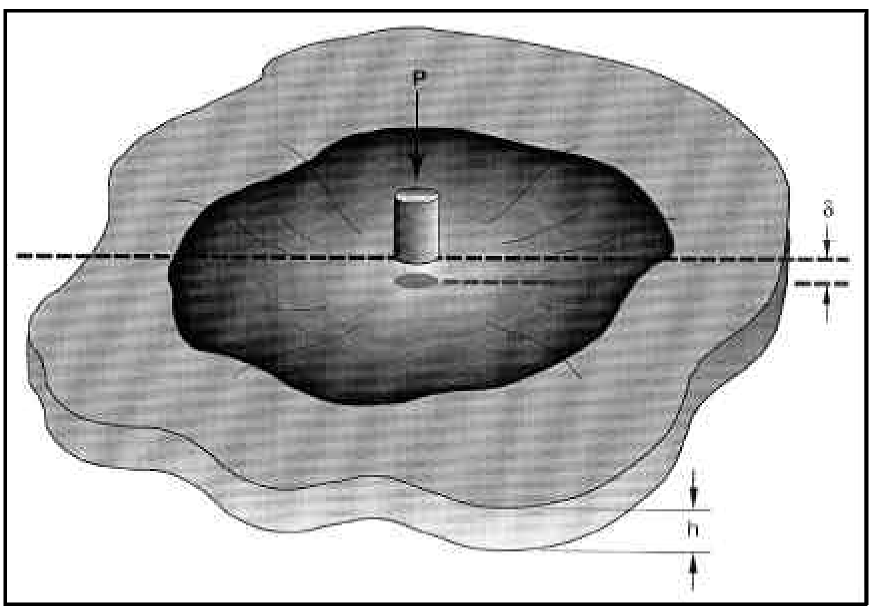
The flexural strength is obtained for the maximum load at failure P' by



a. Cantilever beam method.



b. Simple beam method.



c. Plate deflection method.

Figure 2-1. Determination of the mechanical properties of ice

$$\sigma_b = \frac{3P'L}{2Bh^2}. \tag{2-5b}$$

(3) Plate loading. The characteristic length L_c of ice can be directly measured by loading an ice sheet with a load P (e.g., a drum that can be filled with water from a safe distance away) and measuring the corresponding plate deflection δ as near the edge of load application as possible, as shown in Figure 2-1c.

$$L_c = \sqrt{\frac{P}{8\gamma\delta}} \tag{2-6}$$

where γ is the specific weight of water. For Equation 2-6 to be valid, the ratio of the radius a of the load area to the characteristic length should not exceed 0.2, i.e.

$$\frac{a}{L_c} \approx \frac{a}{17h} \le 0.2. \tag{2-7}$$

Stated another way, a should not exceed 3 to 4 ice thicknesses.

2-4. Frazil Ice

Frazil ice is formed in turbulent, supercooled water. Supercooled water is at a temperature below its equilibrium freezing point; for pure water the freezing point is, by definition, 0°C (32°F) at atmospheric pressure. Supercooling takes place in lakes and rivers in turbulent, open-water areas when the air temperature is significantly less than 0°C (32°F). Usually an air temperature of –8°C (18°F) or lower is required. The requirement for open water can be understood by noting that, if the water surface is covered with ice, the temperature at the ice/water interface must be at the equilibrium freezing point, and all heat transfer from the water would stop when the water cooled to 0°C (32°F). As a result, the formation of frazil is always associated with open water. The level of supercooling should not be overestimated; it will usually not exceed several hundredths of a degree Celsius, and in no case will it exceed 0.1°C (0.18°F). As a result, supercooling is detectable only with laboratory-grade thermometers. Frazil ice appears first as small crystals (0.1 millimeter to several millimeters, 0.004 inch to one or two tenths of an inch) that are distributed more or less uniformly throughout the region of turbulence. In rivers, for example, this is likely to be throughout the entire depth. Each crystal starts out as a perfect disk, whose major diameter is 10 to 12 times its thickness. This disk shape is the form by which frazil is chiefly known, but, with time, frazil ice evolves through a number of processes to form larger and larger ice masses. Eventually, it will become a stationary, floating ice cover that may be many kilometers (miles) long.

a. Frazil ice formation. During the formation stage, the initial frazil ice crystals are created. Formation is characterized by supercooled water, turbulent flow, the rapid growth of disk-shaped crystals, and the creation of new crystals by secondary nucleation (explained below). The length scales of the ice associated with this stage range from several micrometers to perhaps a few millimeters (i.e., between a hundredth and a tenth of an inch). This stage usually takes place during cold periods when the heat loss from the open water surface is intense.

(1) Nucleation. It is now known that the frazil crystals do not "spontaneously" appear through nucleation in the water column. Nucleation is a general term, referring to the formation of a new phase of a substance from a parent phase. In our case, the parent phase is obviously water, and the new phase that is appearing is, of course, ice. We know now that frazil ice crystals are formed from s*eed crystals*, which are ice crystals introduced from outside the natural water body. Seed crystals can come from a number of different sources: vapor evaporating from the water surface, upon encountering cold air, can sublimate into ice crystals, which fall back onto the water surface and are entrained by the turbulent motion of the flow; small water droplets generated by breaking waves; bubbles bursting at the water surface; splashing; and snow and sleet. But seed crystals are not the whole story.

(2) Secondary nucleation. It is observed that once a very few seed crystals are introduced into turbulent supercooled water, very quickly many new crystals are created through *secondary nucleation*. Collisions of existing crystals with hard surfaces (including other crystals) is thought to be the main mechanism through which new frazil ice crystals are formed. These new crystals can then further increase the rate of secondary nucleation with a multiplicative effect. Because the frazil ice crystals are suspended in supercooled water, they are also growing in size. The water temperature will dynamically reflect the balance of the latent heat released by the growing crystals and the heat transfer from the water surface. Eventually, the rate of latent heat released is enough to return the water temperature to the ice–water equilibrium temperature (0°C [32°F]).

b. Frazil ice evolution and transport. Frazil ice evolves and is transported after it forms. Frazil evolves in form largely from the individual crystals joining together to form larger masses. The evolution of frazil is characterized by water more or less at the equilibrium temperature, and frazil in the form of *flocs, anchor ice,* and *floes*. The length scales of the ice associated with this stage range from several millimeters to many meters (i.e., a fraction of an inch to tens of feet). The frazil is largely moving under the influence of the flow velocity of the river or stream, generally at the surface. After cold nights, it is typical to see *frazil slush*, formed of frazil flocs, moving along at the water surface of northern rivers and streams. This ice may travel long distances, moving for many days and may eventually form large moving floes. Eventually, through a process termed *juxtaposition*, the frazil floes may form stationary, floating ice covers of uniform thickness that may be quite large and last for the entire winter season. Other configurations of ice covers may form by a variety of mechanisms, depending on the dynamics of the frazil ice upon its arrival at the site of stationary ice, and depending on the hydraulic conditions at the cover's leading edge. Frazil slush and floes may be entrained in the flow beneath the initial cover to form thicker accumulations. In extreme cases, these floating covers can become very thick, as in the relatively rare instance where a *hanging dam* forms. Frazil ice may be deposited on or eroded from the underside of the cover throughout the winter. The crystal structure of ice covers formed from frazil ice reflects its origin, and the ice crystals tend to be small and randomly oriented.

c. Problems caused by frazil ice. Frazil ice can cause a number of problems. If areas of streams or channels remain open for long periods during cold weather, large amounts of frazil ice can be formed, carried downstream by the flow velocity, and eventually deposited in a relatively slow velocity reach of the river to form a *freezeup ice jam*. Freezeup ice jams can block substantial portions of the river cross section. This blockage may raise upstream water levels enough to cause flooding, or may serve as the site of a *breakup ice jam* later in the winter season. Upstream water levels may also be raised if large amounts of frazil ice are deposited on the channel bottom as *anchor ice* to form *anchor ice dams*. Anchor ice dams are relatively rare, and usually occur in steep, shallow rivers and streams. Water intakes can experience significant problems with frazil ice if they are operated when the water is supercooled. The crystals in the supercooled water will be growing in size and will stick to any object they contact—including intake trash racks—as long as these objects are at a temperature below freezing. Given the effective heat transfer rates provided by flowing water, any object in the water that is not heated will quickly be at the temperature of the supercooled water and will accumulate frazil. Sufficient frazil can accumulate on the trash rack to effectively block it and completely stop the flow of water into the intake, often with severe consequences.

d. Control of frazil ice. An intact, stable ice cover will always prevent the production of frazil ice by "insulating" the water surface and preventing the large heat loss rates responsible for supercooled water. If an ice cover can be successfully created and kept in place over a reach of a river that is normally open during periods of cold weather, frazil ice problems can be completely avoided or substantially reduced. The techniques for creating and maintaining a stable ice cover are described in Chapter 3. Another technique for preventing supercooled water is to mix "warm" water with the supercooled water and raise the water temperature to the ice–water equilibrium temperature, or slightly above. This technique is especially effective near water intakes, where the quantity of warm water required can be modest. Finally, if the actual production of frazil ice cannot be controlled, mechanical removal of the frazil, using techniques described elsewhere in this manual, may be the only recourse.

2-5. Thermal Ice Growth

a. Static ice formation. Ice formation on water in which the flow velocity plays no role is called static ice formation. This includes ice formed on lakes and ponds during periods of low winds, and on rivers and streams in which the flow velocity is approximately 0.3 m/s (1 ft/s) or less. Static ice formation starts in a very thin layer of supercooled water at the water surface and is probably initiated by the introduction of seed crystals. The ice grows at the ice/water interface as a result of heat transfer upward from the interface, through the ice, to the atmosphere. Ice grows in hexagonal crystals with three *a* axes of symmetry in what is called the basal plane, and one *c* axis perpendicular to the basal plane. The orientation of the ice crystals in a static ice cover can vary, depending on the initial formation process. However, once an initial ice cover is formed, continued thermal growth of the initial ice crystals tends to favor the development of vertical *c* axes. Often, ice crystals in a static ice cover look like pencils with the "c" axes as the leads, and are called *columnar*. Because the impurities in the water are "pushed" to the boundaries of columnar crystals during growth, a relatively high concentration of impurities is trapped between the crystal boundaries. Owing to the trapped impurities, melting begins at the crystal boundaries during warm periods and a phenomenon called *candle ice* often develops. In candle ice, innumerable single crystals are no longer frozen together, but rather are leaning on each other for support. A small impact, such as a wave, or a well-placed kick, can collapse the entire mass. Another form of ice found during static ice growth results from the presence of a snow cover on the ice and is called *snow ice*. Snow ice is formed when the weight of a snow cover on the ice sheet is sufficient to depress the ice and cause water to flood up through cracks and saturate the lower layers of the snow. Snow ice is granular, opaque, and white, and it has small, randomly oriented crystals.

b. Thermal balance of ice covers. The thermal balance of ice covers is found by summing all the modes of heat transfer between the ice cover and the atmosphere, and between the ice cover and the water below (Ashton 1986). One important aspect of the thermal heat balance is the heat input through solar radiation (sunlight), especially in the spring, when the hours of daylight are increasing. The ratio of the reflected sunlight to the incident sunlight is defined as the *albedo* of the surface. (An albedo of unity indicates that all of the solar radiation is reflected, while an albedo of zero means that all of the radiant energy is absorbed.) Ice covers that look "white" tend to have high albedos. For example, an ice surface covered with fresh snow can have an albedo of 0.9; ice covers composed of snow ice can have albedos as high as 0.6 to 0.8. In contrast to this, ice covers that are composed of clear columnar ice ("black" ice) may have albedo values as low as 0.2. An attractive and relatively easy way to modify the thermal balance of ice, especially to promote the melting and weakening of the ice cover to reduce the threat of ice jam flooding, is to decrease its albedo by applying a dark material or *dust* to the top surface to increase the absorption of solar radiation. Depending on the type of dust used and amount applied, the albedo can be reduced to 0.15 or 0.2.

c. Estimating thermal ice growth. Predicting the thickness of a natural ice cover attributable to thermal growth is a classic problem of ice engineering. The differential equation describing the thermal growth rate can be formulated by assuming the following:

- That the ice is a homogenous, horizontal layer.
- That the ice is growing only at its horizontal interface with the water.
- That the thermal conditions in the ice are quasi-steady.
- That the heat flux from the water is negligible.
- That the heat fluxes are in the vertical direction only.
- That the heat loss rate from the ice surface to the atmosphere is a linear function of the temperature difference between the ice surface and the air.

Under these assumptions, the heat transfer rate through the ice cover to the atmosphere is equivalent to that of a steady heat flux through a composite slab. The thermal growth rate of the ice is found as

$$\frac{\partial h}{\partial t} = \frac{1}{\rho\lambda} \frac{\left(T_m - T_a\right)}{\left(\dfrac{h}{k_i} + \dfrac{1}{H_{ia}}\right)} \tag{2-8}$$

where

h = ice thickness

T_m = temperature at the water/ice interface (assumed to be the ice–water equilibrium temperature, or 0°C [32°F])

t = time

T_a = air temperature

k_i = thermal conductivity of the ice

H_{ia} = heat transfer coefficient from the ice surface to the atmosphere

ρ = ice density

λ = ice latent heat.

Although Equation 2-8 is nonlinear, it is readily solved to yield the following "standard" model of ice thickness as a function of air temperature

$$h_j = \sqrt{(B + h_k)^2 + 2A(U_j - U_k)} - B \tag{2-9}$$

where

h_j = calculated ice thickness on day *j*

h_k = ice thickness on day *k*, either observed or calculated (note that $j > k$, meaning that day *j* occurs after day *k*).

$$A = \frac{k_i}{\rho\lambda}$$

$$B = \frac{k_i}{H_{ia}}$$

$$U_j = \sum_{i=1}^{j} (T_m - T_{ai})$$

$$U_k = \sum_{i=1}^{k} (T_m - T_{ai})$$

U_j = Accumulated Freezing Degree-Days (AFDDs) recorded between the onset of freezeup (day 1) and day *j*

U_k = AFDDs recorded between the onset of freezeup and day *k* (note that $U_j \geq U_k$).

If the heat conduction through the ice cover is the controlling rate in the overall energy flux, then *B* can be ignored and, if the initial ice thickness is assumed to be zero, then the classic result is found

$$h_j = \alpha\sqrt{U_j} \tag{2-10}$$

where

$$\alpha = \sqrt{\frac{2k_i}{\rho\lambda}} .$$

Typical values for α are presented in Table 2-1. In this case the ice thickness is proportional to the square root of the accumulated freezing degree-days.

d. Remarks. It is not surprising that, for natural ice covers, the assumptions upon which the standard model is based may not always hold true, and other processes, not included in the standard model, may also influence the thermal growth rate of the ice. For example, the presence of snow on the

Table 2-1
Typical Values of α (after Michel 1971)

Ice Cover Condition	α*	α†
Windy lake w/no snow	2.7	0.80
Average lake with snow	1.7-2.4	0.50-0.70
Average river with snow	0.4-0.5	0.12-0.15
Sheltered small river	0.7-1.4	0.21-0.41

* AFDD calculated using degrees Celsius. The ice thickness is in centimeters.
† AFDD calculated using degrees Fahrenheit. The ice thickness is in inches.

ice cover may influence the heat transfer rate from the ice surface to the atmosphere. In theory, this influence could be accounted for in the standard model if the snow depth and the snow thermal conductivity were known. The ice surface may also be flooded if the weight of the accumulated snow is greater than the buoyant force of the ice cover. This will cause part of the snow to become saturated by water flowing upward through cracks in the ice. This saturated snow is able to freeze relatively rapidly, forming snow ice. In addition, the heat flux from the ice surface to the atmosphere is composed of several modes of heat transfer, including shortwave radiation, longwave radiation, evaporation, and sensible heat loss. The actual heat transfer rate is only approximated by the relationship included in the standard model. As a result, H_{ia}, the heat transfer coefficient, may not be a constant but may vary with the meteorological conditions. Given all this, however, the standard model represented by Equation 2-9 still represents a good, practical model of ice growth. To go beyond the standard model requires an extensive data collection, and, to date, there has been no indication that the additional effort would be rewarded by a more accurate model.

2-6. Dynamic Ice Cover Formation

When an ice cover's growth is dominated by the interaction between the transported ice pieces and the flowing water, the cover is said to form dynamically. This is the counterpart of the thermal formation and growth described earlier. Almost all river ice covers are formed dynamically. All ice covers that form in this way progress upstream from an initiation point as ice is brought to the leading upstream edge of the ice cover by the flow of the river. Many different and separate processes may occur at the leading edge, depending on the hydraulic flow conditions and the form of the arriving ice. The various processes at the leading edge are described in a general way in the following.

a. Bridging. At very low flow velocities and relatively high concentrations of surface ice, it may be possible for the ice cover to spontaneously arch across the open width of the channel and stop moving. It is generally not possible to predict where these bridging locations will be without historical knowledge. To assure the initiation of an ice cover at a specific location, ice control booms or hydraulic control structures, or both, may be necessary.

b. Juxtaposition. At relatively low flow velocities, ice floes arriving at the leading edge may simply come to a stop and not underturn. In this way the ice cover will progress upstream by juxtaposition. The maximum flow velocity at which juxtaposition happens depends on the floe geometry and the channel depth. Generally, ice control booms will function properly only if juxtaposition of the arriving ice is possible.

c. Underturning of floes. At higher flow velocities, the arriving floes may not be stable but may instead underturn. If the flow velocity is not too high, these underturned floes will remain at the leading edge of the ice cover.

d. Ice cover shoving. Shoving in the ice cover can happen over a wide range of flow velocities. The cover collapses in the downstream direction and becomes thicker if the forces acting on it exceed its ability to withstand those forces. The strength of an ice cover formed from many separate pieces of ice increases with its thickness, so that when shoving takes place, the strength of the ice cover is increased. An ice cover may repeatedly shove and thicken as it progresses upstream. If the ice cover is treated as a "granular" material, its strength characteristics and its final thickness can be estimated.

e. Under-ice transport of floes. At relatively high flow velocities, the ice floes arriving at the leading edge of the ice cover may be underturned and transported under the ice cover for considerable distances. At this point, further upstream progression may be halted until the deposition of the floes somewhere downstream of the leading edge reduces the channel conveyance sufficiently to cause the upstream water levels to rise and the flow velocities at the leading edge to be reduced.

f. No ice cover progression. The ice cover will stop progressing upstream if the flow velocities at the leading edge remain too high. In this case open water will remain upstream of the leading edge throughout the winter season. This will result in the production of frazil ice in the open-water area all winter, which may lead to the formation of freezeup jams or other problems downstream.

2-7. Ice Cover Breakup

Breakup transforms a completely ice-covered river into an open river. Two extreme forms of breakup bracket the types of breakup commonly found throughout most of North America. At one extreme is *thermal meltout*. During an ideal thermal meltout, the river ice cover deteriorates through warming and the absorption of solar radiation and melts in place, with no increase in flow and little or no ice movement. At the other extreme is the more complex and less understood *mechanical breakup*. Mechanical breakup requires no deterioration of the ice cover but rather results from the increase of river discharge. The increase in flow induces stresses in the cover, and the stresses in turn cause cracks and the ultimate fragmentation of the ice cover into pieces that are transported by the channel flow. Ice jams take place at locations where the ice fragments stop. Severe and sudden flooding can result when these ice jams form or when they release. Actual breakups take place most often during warming periods, when the ice cover strength deteriorates to some degree and the flow in the river increases because of snowmelt or precipitation. Therefore, most river ice breakups actually fall somewhere in between the extremes of thermal meltout and mechanical breakup. As a general rule, the closer that a breakup is to being a mechanical breakup, the more dramatic and dangerous it is because of the increase in flow and the large volume of fragmented ice produced.

a. Thermal meltout. Every river in North America will experience a thermal meltout every spring unless a mechanical breakup occurs first. Thermal meltouts will not take place at all points on a river simultaneously, but will occur at different locations at different rates depending on the latitude, local climate, and ice exposure. Thermal meltouts happen because of heat transfer into the ice cover by convection to its underside from the water, by convection from the warm air to its top, and radiation, both longwave (infrared) and shortwave (sunlight). The transfer of heat from the water to the underside of the ice cover can be very substantial, especially if there is open water upstream that provides an area in which the flowing water can absorb heat from the atmosphere. In almost all cases, the albedo of open-water areas will be much less than the albedo of the ice cover. As a result, the open-water areas will absorb more solar radiation than ice-covered areas. When the flowing water passes under the ice cover, a portion of the extra heat provided by this lower albedo will be available to melt its underside. Generally, the albedo of snow on the ice surface or snow ice will be quite large and little solar radiation will penetrate the ice cover. The creation of meltwater on the surface will drastically lower the albedo and help the ice absorb sunlight. The ice cover can also deteriorate internally without much of a loss of thickness if solar radiation is able to penetrate it. The absorbed solar radiation causes melting in the interior of the ice that results in a loss of structural integrity of the cover, as described previously in paragraph 2-5a. This is most likely to happen if the ice cover is composed of columnar crystals. Fine-grained ice covers, composed of snow ice or frazil ice, are much less susceptible to internal deterioration through absorption of solar radiation.

b. Mechanical ice cover breakup. Breakup does not happen simultaneously everywhere along a river network. Often breakup occurs first on smaller tributaries, and then proceeds haphazardly to the main stem rivers. This can result in severe ice jams at the confluence of tributaries and the main stems. Breakup can progress upstream or downstream, depending on the local weather and the flow direction of the river. On rivers that flow north, or flow from a warmer area to a colder area, breakup often progresses from upstream to downstream. Generally, if downstream locations release their ice first, fewer ice jams will result than if the breakup front progresses from upstream to downstream. Every breakup is different, but there are a few broad similarities in the sequence of a breakup that can be described. The mechanical breakup always occurs in response to an increase in flow in the river, with a corresponding increase in stage.

(1) Formation of shore cracks. Shore cracks are longitudinal cracks running parallel to the banks of the rivers. Shore cracks form as a result of changes in water level. Controlling factors are the material properties of the ice, ice thickness, channel width, and the type of attachment of the ice cover to the channel bank (hinged or fixed). Only a small increase or decrease in discharge is necessary to cause shore cracks, and they are usually common soon after runoff into a river has begun to increase. The presence of shore cracks does not necessarily indicate the immediate onset of breakup. They may be present throughout the winter season.

(2) Cracking of the ice sheet into individual floes. Transverse cracks (across the channel) will appear soon after the river stage has begun to increase. The first cracks will generally create relatively large ice floes, a river-width wide, and many river-widths long, but sometimes the ice covers are immediately broken into much smaller floes. The actual mechanisms responsible for creating the individual floes have not yet been positively identified.

(3) Movement of floes. As the stage continues to increase, the ice floes will begin to move. If the floes are relatively large, they may be held in place by sharp bends, constrictions, bridge piers, etc., until a substantial increase in stage is reached. If the floes are relatively small, and there are no constraints, they may begin to move after a small stage increase. As a rule of thumb, the stage must rise 1-1/2 to 3 times the ice thickness before the ice moves. Once the floes begin moving, they are quickly reduced in size, eventually attaining a diameter that is roughly 4 to 6 times the ice thickness.

(4) Formation of ice jams. Ice jams form when the moving ice floes reach a location in the river where its ice transport capacity is exceeded. This is most likely at places where an intact ice cover remains, the slope of the river decreases, a geometric constraint exists, etc. At these locations, the ice stops moving and jams. This type of ice jam is a *breakup ice jam*. Ice jams substantially reduce the channel flow conveyance. As a result, water levels upstream of an ice jam can rise substantially and quickly, causing flooding and transporting ice into the floodplain. The probable maximum thickness and roughness of ice jams can be estimated and used to estimate the probable flood stages (see Chapter 4).

2-8. References

a. Required publications.
None.

b. Related publications.

Ashton 1986
Ashton, G.D., ed. 1986. *River and Lake Ice Engineering*, Water Resources Publications, Littleton, Colorado.

Frankenstein 1968
Frankenstein, G.E. 1968. *Strength of Ice Sheets*, National Research Council of Canada, Associate Committee on Geotechnical Research, Technical Memorandum No. 92, pp. 79–87.

Gow 1977
Gow, A.J. 1977. "Flexural Strength of Ice on Temperate Lakes," *Journal of Glaciology*, Vol. 19, No. 81, pp. 247–256.

Heggen 1983
Heggen, R.J. 1983. "Thermal Dependent Properties of Water," *Journal of Hydraulic Engineering*, American Society of Civil Engineers, Vol. 109, No. 2, pp. 298–302.

IAHR 1980
International Association for Hydraulic Research. 1980. "Standardization of Testing Methods for Ice Properties," *Journal of Hydraulic Research*, Vol. 18, No. 2, pp. 153–165.

Lavrov 1969
Lavrov, V.V. 1969. *Deformation and Strength of Ice*, Translation TT 70 50130, National Science Foundation, Washington, D.C.

Michel 1971
Michel, B. 1971. *Winter Regime of Rivers and Lakes*, Monograph III-B1a, U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire.

Michel 1978
Michel, B. 1978. *Ice Mechanics*, Les Presses de l'Université Laval, Québec, Canada.

Pounder 1965
Pounder, E.R. 1965. *The Physics of Ice*, Pergamon Press Ltd., Oxford, England.

Sodhi et al. 1985
Sodhi, D.S., C. Martinson, and W.B. Tucker III. 1985. "Determining the Characteristic Length of Floating Ice Sheets by Moving Loads," *Proceedings, Fourth International Offshore Mechanics and Arctic Engineering Symposium*, Dallas, Texas, 17–21 February 1985, Vol. 2.